Relativistic orbits around a massive black hole

The statistical mechanics approach

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Summary

We demonstrate how the complicated dynamics of a nearly Keplerian N-body system can be described and studied in a formal statistical mechanics framework, where the background potential is described as a correlated Gaussian noise. We show that the evolution of the angular momentum of stars, which rapidly precess due to general relativity, depends critically on the temporal smoothness of this noise. A Schwarzschild precession induced barrier in angular momentum emerges when the noise is sufficiently smooth.

Motivation

Stars around a massive black hole (MBH) move on nearly fixed Keplerian orbits. The random fluctuations of the discrete stellar background cause small potential perturbations, which accelerate the evolution of angular momentum by resonant relaxation. This drives many phenomena near MBHs, such as extreme mass-ratio gravitational wave inspirals, the warping of accretion disks, and the formation of exotic stellar populations.

Methods

- ▹ We developed a formal statistical mechanics framework to analyze such systems.
- ▹ We described the stellar potential as a correlated Gaussian noise.
- ▹ We derived the leading order, stochastic equations of motion.
- ▹ We obtained an effective Fokker-Planck equation for a general correlated Gaussian noise.

Fokker-Planck description

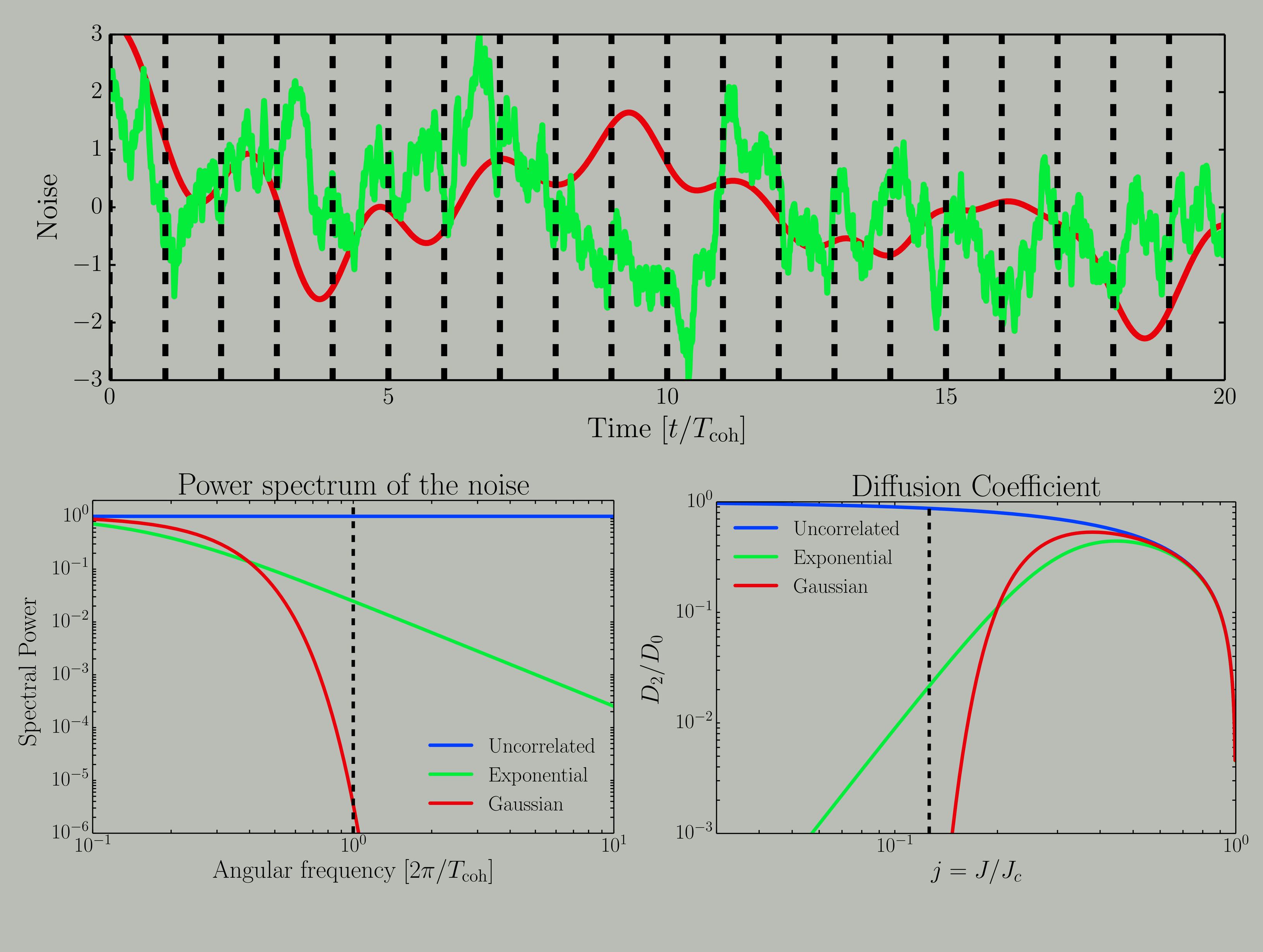
Fokker-Planck equation:

$$\frac{\partial}{\partial t}P(j,t)=\frac{1}{2}\frac{\partial}{\partial j}\left\{jD_2(j)\frac{\partial}{\partial j}\left[P(j,t)/j\right]\right\}$$

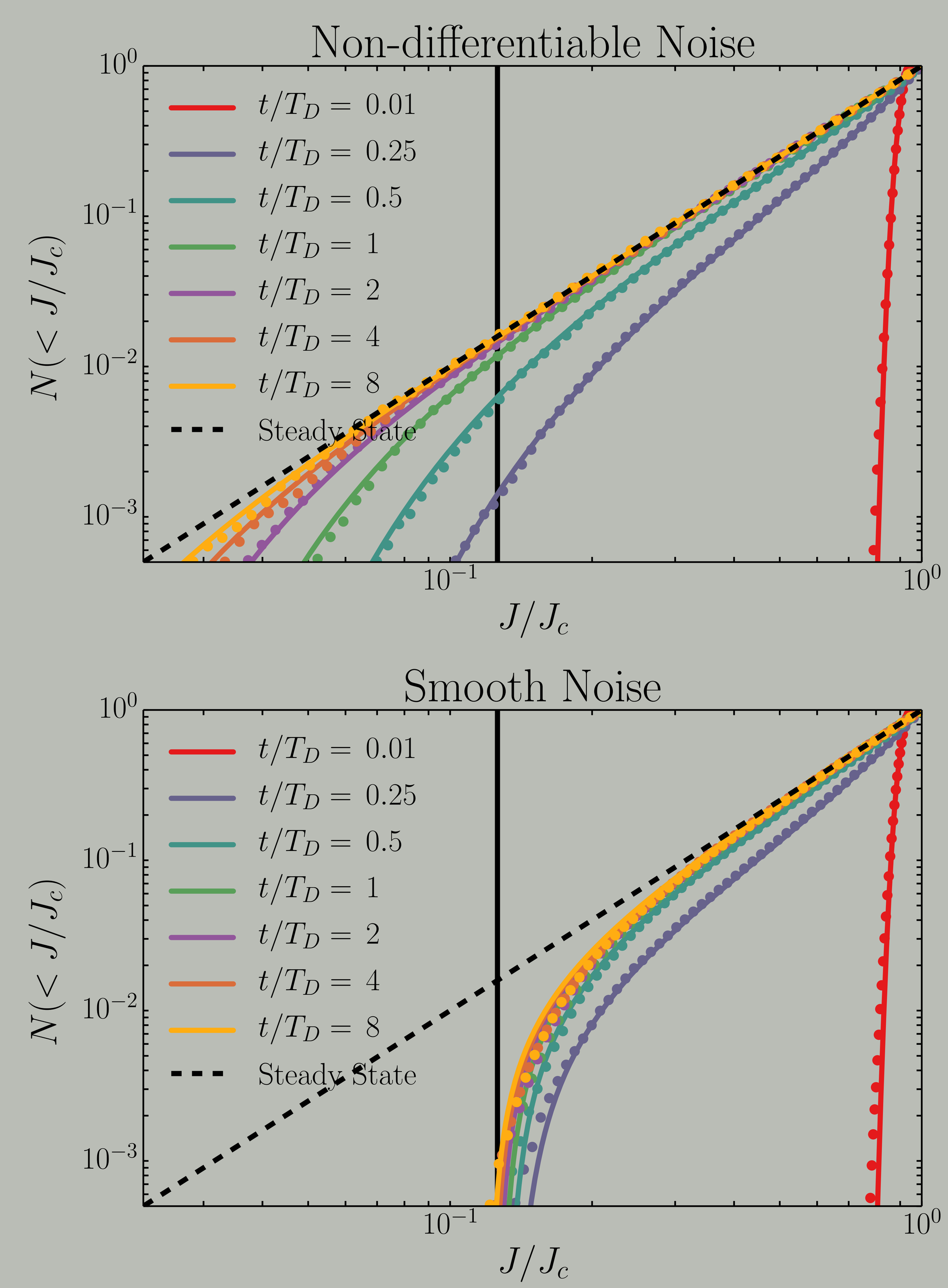
The diffusion coefficient is proportional to the power spectrum of the noise $S_\eta(\nu)$ at the precession frequency ν_p:

$$D_2(j)=\nu_j^2(j)\,S_\eta(\nu_p(j))=\nu_j^2(j)\,\mathcal{F}\left[C(t)\right](\nu_p(j))$$

Noise models and corresponding diffusion coefficients

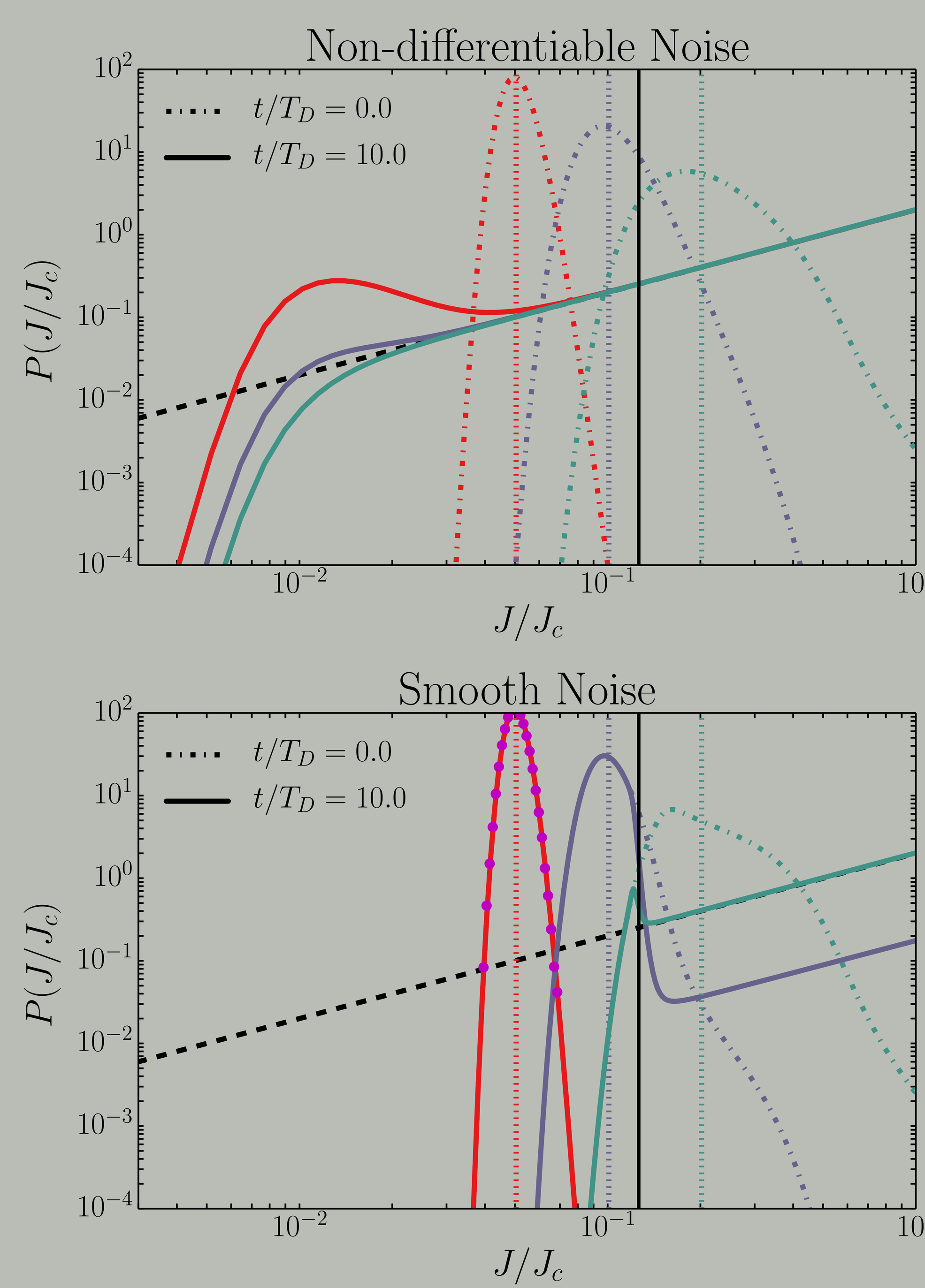


Relativistic induced barrier in phase space



- ▹ The evolution of angular momentum depends critically on the temporal smoothness of the background potential (noise).
- ▹ Smooth noise has a maximal variability frequency $\nu_{\max}$ beyond which there is a vanishing power in the noise and the evolution of the angular momentum is exponentially suppressed.
- ▹ Therefore a relativistic test star, undergoing Schwarzschild (in-plane) General Relativistic precession with frequency ν_{GR}/j^2, is exponentially suppressed for $j<j_b$, where $\nu_{GR}/j_b^2\sim\nu_{\max}$, due to the adiabatic invariance of the precession against the slowly varying random background torques.
- ▹ This results in an effective Schwarzschild precession-induced barrier in angular momentum, which exist only if the noise is smooth.

Time evolution of high eccentricity orbits



- ▹ The evolution of the PDF strongly depends on the noise model and on the location of the initial angular momentum, j_i.
- ▹ For a smooth noise, test stars with initial $j_i\gg j_0$ reach the equilibrium maximal entropy distribution by time $t\approx T_D$, while stars starting at $j_i\ll j_b$ remain out of equilibrium on times $t\gg T_D$ near their initial distribution.

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